**Teacher notes**

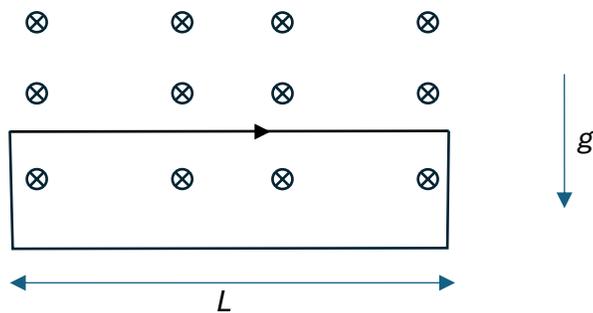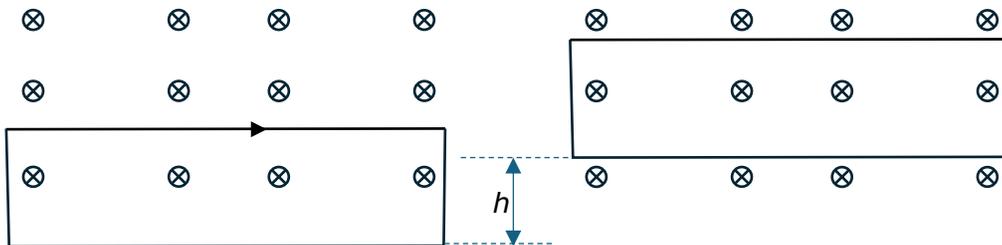**Topic D**

## Does the magnetic force do work?

A length $L$ of a wire carrying current $I$ is in the region of a uniform horizontal magnetic field $B$. The weight of the wire is $W$.



(a) Determine the current so that the magnetic force balances the weight of the wire.
(b) The current is increased above the value found in (a). The magnetic force will raise the wire.



Discuss this situation from the point of view of work done when the wire has been raised by a distance $h$.

## Answers

(a) The upward magnetic force is $BIL$ and so $BIL = W$ which gives $I = \dfrac{W}{BL}$.

(b) As the wire moves up an electron in the wire acquires a vertical velocity component and so the magnetic force on the electron is no longer vertical. It is at right angles to the velocity and so is rotated to the right as shown below.
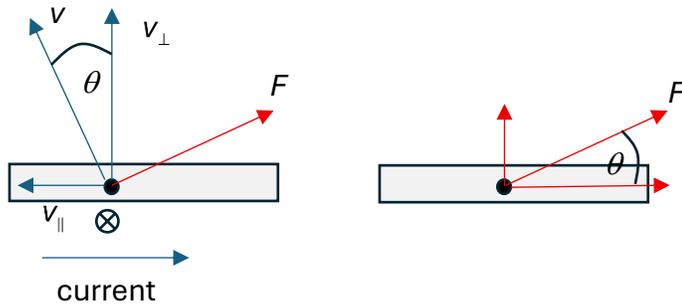


Suppose that the charge per unit length is $k$. Then the total charge in a length $L$ of the wire is $kL$.

But also $I = kv$ where $v$ is the speed of the electrons. To show this: in time $\delta t$ an amount $\delta Q = kv\delta t$ of charge went past the cross sectional area of the wire. The current is then $I = \dfrac{\delta Q}{\delta t} = kv$.

This means that

$$F_\parallel = F\cos\theta \qquad\qquad\qquad F_\perp = F\sin\theta$$

$$= BIL\frac{v_\perp}{v} \qquad\qquad\qquad\qquad = BIL\frac{v_\parallel}{v}$$

and similarly

$$= BLkv\frac{v_\perp}{v} \qquad\qquad\qquad\qquad = BLkv\frac{v_\parallel}{v}$$

$$= kBLv_\perp \qquad\qquad\qquad\qquad\quad = kBLv_\parallel$$

(These results are obvious if you know the vector product.)

The horizontal component of the force acts in a direction opposite to the electron's velocity. In time $\delta t$ the electron moves a horizontal distance $v_\parallel \delta t$ and so the work done on the electrons is

$$\delta W = F\delta r$$
$$= kLBv_{\perp} \times v_{\parallel}\delta t$$
$$W = kLBv_{\parallel} \times \int v_{\perp}\, dt$$
$$= (kv_{\parallel})LB\int v_{\perp}\, dt$$
$$= ILB\int v_{\perp}\, dt$$

This means $W = BILh$ since $\int v_{\perp}\, dt = h$.

(Here the current is the difference between the actual current in the circuit and the current found in (a).)

This is the puzzle: the work done is $W = BILh = F_{\perp}h$ where $F_{\perp}$ is the normal component of the magnetic field. It **looks** like this component of the magnetic force on the wire is doing the work by lifting the wire. But this cannot be the case since the magnetic force cannot do work on a moving particle (the force is perpendicular to the velocity). The only possibility is that this work was done by the battery supplying the current. The motion of the electrons is being opposed by the magnetic force, so the battery has to supply the work to keep the current constant in the circuit.

This approach ignores electromagnetic induction effects so it is valid when the wire moves upwards very slowly so that the rate of change of flux, and hence the induced emf, is small.